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Nonlinear Disturbance Rejection for Magnetic Levitation Systems[1]

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Abstract: *In this paper, a position regulation control strategy is developed for a magnetic levitation system operating in the presence of a bounded, nonlinear, periodic disturbance. The proposed controller utilizes a saturated control force input in conjunction with a learning based disturbance estimator to asymptotically regulate the target mass to a desired set point position despite the actuator's unidirectional limitation of exerting only an attractive force on the target mass (i.e., the control input can only generate an attractive force while the earth's gravitational field is utilized to produce the repulsive action). In addition, the control development only requires the disturbance input to be bounded and periodic in nature. Simulation results are included to illustrate the performance of the control strategy.*

1 Introduction

With ever increasing demands placed on precision and reliability within manufacturing and research environments, magnetic levitation systems are finding increased utilization within such applications as machine tooling due to their non-contact (*i.e.,* low friction) force exertion and their capability for active attenuation of mechanical vibrations. However, magnetic levitation systems suffer from various control complexities such as: i.) open-loop instability, ii.) inherent nonlinearities within the system model, iii.) a unidirectional force input, and iv.) continuous biasing. In response to these difficulties, researchers have employed various modelling and control development techniques to address the aspects of i.) through iii.). For example, Charara *et al.* [2] constructed a nonlinear model of an inertial wheel supported by active magnetic bearings, for which a sliding mode controller was then designed to stabilize the system. In [7], Lévine *et al.* proposed a nonlinear feedback control law for the positioning of a shaft based on the current complementarity or current almost complementarity condition. Queiroz *et al.* [4] utilized a nonlinear model of a planar rotor disk, active magnetic bearing system to develop a global exponential position tracking controller for the full-order electromechanical system. In [8], Mohamed *et al.* demonstrated that the Q-parameterization theory could be utilized to autobalance the rotor of a vertical shaft active magnetic bearing system.

Recently, much effort has been directed toward the area of disturbance suppression within magnetic levitation systems. For example, in [3], Costic *et al.* introduced a learning based controller to asymptotically regulate a magnetic bearing system while compensating for periodic, exogenous disturbances. Rodrigues *et al.* [9] proposed an interconnection and damping assignment Passivity-Based controller to yield a smooth stabilizing controller for the active magnetic bearing system. In [6], Gentili utilized a model-based regulation approach to achieve set point regulation of the target with the disturbance input being modeled as the sum of a finite number of sinusoids. Similarly, Behal *et al.* [1] designed a set of linear, bounded-input bounded-output filters to facilitate the utilization of standard adaptive techniques that compensated for an unknown sinusoidal disturbance signal. Though not directly targeted at magnetic levitation systems, Xian *et al.* [11] proposed an adaptive disturbance rejection approach for single-input single-output, linear time invariant, uncertain systems subjected to sinusoidal disturbances with unknown amplitude and frequency. The approach of [11] utilizes a state estimate observer in a back stepping fashion with only output measurements to achieve asymptotic disturbance rejection.

In this paper, the topic of disturbance rejection within the magnetic levitation area is furthered pursued by designing a saturated force control input that achieves asymptotic target position regulation despite the presence of a *nonlinear*, bounded, periodic disturbance. The control development differs from previous disturbance rejection controllers in that restrictions on the explicit structure of the disturbance signal are not required (*i.e.,* the disturbance force need only be bounded and periodic). As with previously designed magnetic levitation controllers, the proposed control structure must contend with the constraint that the actuation force is unidirectional. That is, the magnetic actuator can only exert an attractive force on the target mass (the earth's gravitational field is utilized to produce the repulsive action).

The remainder of the paper is organized as follows. The model of a magnetic levitation system actuating on a target mass suspended in the gravitation field is presented in Section II. Section III identifies the control objectives and constraints of the control development. The saturated control force input and the learning based disturbance estimator are presented in Section IV. A Lyapunov stability analysis is utilized to illustrate the asymptotic regulation of the target position. Simulation results are presented in Section V.

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14. ABSTRACT
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2 System Model

A magnetic levitation system consisting of a target mass suspended vertically in the gravity field subjected to a nonlinear periodic disturbance force can be modelled by the following dynamics [6]

$$\ddot{x} = -g + u^2 + \delta(t) \tag{1}$$

where $x(t)$, $\dot{x}(t)$, $\ddot{x}(t) \in \Re^1$ represent the target mass position, velocity, and acceleration signals, respectively, $g = 9.81\ m/s^2 \in \Re^1$ denotes the gravitational acceleration constant, $u^2(t) \in \Re^1$ represents the control force input[1], and $\delta(t) \in \Re^1$ denotes the nonlinear, periodic disturbance force (note that the system of (1) has been normalized with respect to the target mass m).

Remark 1 *The disturbance force $\delta(t)$ is assumed to be periodic and bounded as given by the following*

$$\delta(t) = \delta(t-T), \qquad |\delta(t)| \leq \delta_o \leq g, \tag{2}$$

where $T \in \Re^1$ denotes the known period and δ_o represents a positive bounding constant.

3 Problem Statement

The control objective is to design a suitable force input that regulates the target mass position to a desired set point in the presence of an unknown, bounded periodic disturbance. The controller design is complicated by the presence of a nonlinear, periodic disturbance and also by the fact that the levitation system can only exert an attraction force on the target mass. In order to facilitate the control development, the target mass tracking error signal $e(t) \in \Re^1$ and the filtered tracking error signal $r(t) \in \Re^1$ are defi nged in the following manner

$$e = x - x_d \qquad r = \dot{e} + \alpha e \tag{3}$$

where $x_d \in \Re^1$ denotes the constant desired set point position and $\alpha \in \Re^1$ represents a positive, constant scalar control gain. The proposed control force is developed under the assumption that the target's position and velocity signals are available for measurement.

4 Saturated Controller Design

The development of the regulation controller is simplified by rewriting the second order system of (1) in terms of the filtered tracking error signal $r(t)$ in the following manner

$$\dot{r} = -g + u^2 + \delta(t) - \alpha^2 e + \alpha r \tag{4}$$

where (1) and the definition of (3) have been utilized. Based on the structure of the ensuing stability analysis, the saturated control force input $u^2(t)$ is designed in the following manner

$$u = \sqrt{g - k_1 \tanh(k_3 r) - \hat{\delta}(t)} \tag{5}$$

where $\hat{\delta}(t) \in \Re^1$ represents a learning based estimate for $\delta(t)$ that is generated on-line via the following expression

$$\hat{\delta}(t) = sat_{\delta_o}\left(\hat{\delta}(t-T)\right) + k_2 \tanh(k_3 r) \tag{6}$$

where the scalar function $sat_{\delta_o}(\cdot)$ is defined in the following manner

$$sat_{\delta_o}(\varepsilon) = \begin{cases} \varepsilon & \text{for } |\varepsilon| \leq \delta_o \\ sgn(\varepsilon)\,\delta_o & \text{for } |\varepsilon| > \delta_o \end{cases} \tag{7}$$

with $\varepsilon \in \Re^1$ representing an arbitrary scalar argument, $sgn(\cdot)$ denoting the standard signum function, and k_1, k_2, $k_3 \in \Re^1$ denoting positive scalar control gains with k_1 and k_2 selected in the following manner

$$k_1 + k_2 \leq g - \delta_o \tag{8}$$

Remark 2 *The structure of (6) and the definition of (7) provide a bounded disturbance estimation signal $\hat{\delta}(t)$ in the sense that*

$$\left|\hat{\delta}(t)\right| \leq \delta_o + k_2;$$

the saturated feedback term $k_1 \tanh(k_3 r)$ in the control input of (5) and the selection of the control gains in (8) ensure that the radicand of (5) is non-negative.

Remark 3 *Based on the definition of the function $sat_{\delta_o}(\cdot)$ in (7), the following inequality can be illustrated (See Appendix A for details)*

$$\left[\delta(t) - sat_{\delta_o}\hat{\delta}(t)\right]^2 \leq \left[\delta(t) - \hat{\delta}(t)\right]^2. \tag{9}$$

Remark 4 *Digital implementation of the learning based, disturbance force estimator $\hat{\delta}(t)$ for slowly periodic disturbance forces (i.e., the periodic interval T is large) may require significant memory allocation if the control sampling frequency is large. That is, the estimation algorithm of (6) requires the logging of exactly one period of the previous estimator values. For example if the controller's sampling frequency is adjusted to $10\ KHz$ and the disturbance force exhibits a period of $T = 1.0$ sec, then the selected processing system would be required to store $(T) \cdot 10^4 = 10{,}000$ data points in memory. Though not a significant concern for memory abundant PC applications, this phenomenon more affects the compact, memory limited DSP or microprocessor based implementations systems.*

After substituting the control force input of (5) into the open loop dynamics of (4) and then cancelling common terms, the closed loop dynamics for $r(t)$ can be formulated in the following manner

$$\dot{r} = -k_1 \tanh(k_3 r) + \left(\delta(t) - \hat{\delta}(t)\right) - \alpha^2 e + \alpha r. \tag{10}$$

Theorem 1 *If the control gains are selected to satisfy the following condition,*

$$\alpha\left(2k_1 + k_2 - \alpha^3\right) \geq \left[|\dot{e}(0)| + \alpha|e(0)| + \frac{1}{2}e(0)^2 + \frac{1}{k_3}\ln(2) + \frac{1}{k_3}\right]^2 \tag{11}$$

[1] Since the contribution of this paper lies within the compensation of the nonlinear disturbance force $\delta(t)$, we have chosen to neglect the electrical dynamics for the magnetic levitation system.

where $e(0)$ and $\dot{e}(0)$ denote the initial target position/velocity tracking error signals, then the control force input of (5) and the disturbance estimator $\hat{\delta}(t)$ of (6) ensure that the target position tracking error $e(t)$ is driven asymptotically to zero in the sense that

$$\lim_{t\to\infty} e(t) = 0. \tag{12}$$

Proof: In order to illustrate the asymptotic regulation of $e(t)$ in (12), the following non-negative scalar function $V(t)$ is defined in the following manner

$$\begin{aligned} V = \; & \frac{1}{k_3}\ln(\cosh(k_3 r)) + \frac{1}{2}e^2 \\ & + \frac{1}{2k_2}\int_{t-T}^{t}\left[\delta(\sigma) - sat_{\delta_0}\left(\hat{\delta}(\sigma)\right)\right]^2 d\sigma. \end{aligned} \tag{13}$$

After taking the time derivative of (13), substituting in the closed loop expression given by (10) and utilizing the definition of (3), the following expression is obtained for the time derivative of $V(t)$

$$\begin{aligned} \dot{V} = \; & \tanh(k_3 r)\left[-k_1\tanh(k_3 r) + \left(\delta(t) - \hat{\delta}(t)\right)\right] \\ & + \left(\alpha r - \alpha^2 e\right)\tanh(k_3 r) + e(r - \alpha e) \\ & - \frac{1}{2k_2}\left[\delta(t-T) - sat_{\delta_0}\left(\hat{\delta}(t-T)\right)\right]^2 \\ & + \frac{1}{2k_2}\left[\delta(t) - sat_{\delta_0}\left(\hat{\delta}(t)\right)\right]^2. \end{aligned} \tag{14}$$

After expanding and simplifying the terms located on the second row of (14), $\dot{V}(t)$ can be rewritten in the following form

$$\begin{aligned} \dot{V} = \; & \tanh(k_3 r)\left[-k_1\tanh(k_3 r) + \left(\delta(t) - \hat{\delta}(t)\right)\right] \\ & + \left(\alpha r - \alpha^2 e\right)\tanh(k_3 r) + e(r - \alpha e) \\ & + \frac{1}{2k_2}\left\{\left[\delta(t) - sat_{\delta_0}\hat{\delta}(t)\right]^2 - \left[\delta(t) - \hat{\delta}(t)\right]^2\right\} \\ & + 2k_2\tanh(k_3 r)\left(\delta(t) - \hat{\delta}(t)\right) \\ & - \frac{1}{2k_2}\left\{k_2^2\tanh^2(k_3 r)\right\} \end{aligned} \tag{15}$$

where (2) and (6) have been utilized. The inequality of (9) can be utilized to upper bound (15) in the following manner

$$\begin{aligned} \dot{V} \le \; & -\left(k_1 + \frac{k_2}{2}\right)\tanh^2(k_3 r) \\ & + \tanh(k_3 r)\left[-\alpha^2 e + \alpha r\right] + er - \alpha e^2. \end{aligned} \tag{16}$$

In order to complete the stability analysis, the two cases when $r(t) = 0$ and $r(t) \neq 0$ are examined.

Case 1: $r(t) = 0$

Based on the fact of $r(t) = 0$, the following expression is obtained

$$\tanh(k_3 r) = 0; \tag{17}$$

thus, the time derivative of $V(t)$ of (16) can be simplified into the following form

$$\dot{V} \le -\alpha e^2. \tag{18}$$

Case 2: $r(t) \neq 0$

Since $r(t) \neq 0$, the following condition exists

$$\tanh(k_3 r) \neq 0. \tag{19}$$

Based on (19), $\dot{V}(t)$ of (16) can be rewritten in the following manner

$$\begin{aligned} \dot{V} \; & \le \; -\left(k_1 + \frac{k_2}{2} - \alpha\frac{r}{\tanh(k_3 r)}\right)\tanh^2(k_3 r) \\ & \quad -\alpha e^2 - \left[\alpha^2 - \frac{r}{\tanh(k_3 r)}\right]\left[e\cdot\tanh(k_3 r)\right] \\ & \le \; -\mathbf{z}^T B\mathbf{z} - \frac{\alpha}{2}e^2 \end{aligned} \tag{20}$$

where $\mathbf{z}(t) \in \Re^{2\times 1}$ represents an auxiliary vector defined by the following

$$\mathbf{z} = \begin{bmatrix} e & \tanh(k_3 r) \end{bmatrix}^T \tag{21}$$

and the matrix $B(t) \in \Re^{2\times 2}$ is defined in the following manner

$$B = \begin{bmatrix} \frac{\alpha}{2} & \frac{\alpha^2}{2} - \frac{r}{2\tanh(k_3 r)} \\ \frac{\alpha^2}{2} - \frac{r}{2\tanh(k_3 r)} & k_1 + \frac{k_2}{2} - \alpha\frac{r}{\tanh(k_3 r)} \end{bmatrix}. \tag{22}$$

In order to ensure that the time derivative of $V(t)$ is always negative or zero, the control gains must be selected in a fashion such that the matrix B of (22) is positive definite; thus, the control gains must be selected to guarantee that the following conditions are satisfied

$$\begin{aligned} & \text{i.)} \quad \frac{\alpha}{2} > 0, \\ & \text{ii.)} \quad \frac{\alpha}{2}\left(k_1 + \frac{k_2}{2} - \alpha\frac{r}{\tanh(k_3 r)}\right) \\ & \qquad - \left(\frac{\alpha^2}{2} - \frac{r}{2\tanh(k_3 r)}\right)^2 > 0. \end{aligned} \tag{23}$$

Clearly from condition i.), the control gain α must be lower bounded by zero in the sense that $\alpha > 0$. Condition ii.) of (23) can be expanded and simplified to obtain the following

$$\text{ii.)} \quad \frac{\alpha}{2}(k_1 + \frac{k_2}{2}) - \frac{\alpha^4}{4} - \frac{r^2}{4\tanh^2(k_3 r)} > 0. \tag{24}$$

After utilizing the fact that [5]

$$\left|\frac{r(t)}{\tanh(k_3 r)}\right| \le |r(t)| + \frac{1}{k_3}, \tag{25}$$

the condition of (24) can be rewritten as followings

$$\text{ii.)} \quad \alpha\left(2k_1 + k_2 - \alpha^3\right) > \left(|r| + \frac{1}{k_3}\right)^2. \tag{26}$$

Based on the structure of $V(t)$ defined in (13), it can be seen that

$$V \ge \frac{1}{k_3}\ln(\cosh(k_3 r)) \ge \left[|r| - \frac{1}{k_3}\ln(2)\right]; \tag{27}$$

hence, the condition of (26) can then be rewritten in the following form

$$\text{ii.)} \quad \alpha\left(2k_1 + k_2 - \alpha^3\right) > \left(V + \frac{1}{k_3}\ln(2) + \frac{1}{k_3}\right)^2. \tag{28}$$

With selection of the control gains according to i.) of (23) and ii.) of (28), the auxiliary matrix B is positive definite; thus, $\dot{V}(t)$ of (16) can be upper bounded by the following expression

$$\dot{V} \leq -\frac{\alpha}{2}e^2 \quad \text{for} \quad \alpha\left(2k_1 + k_2 - \alpha^3\right) > \left(V + \frac{1}{k_3}\ln(2) + \frac{1}{k_3}\right)^2. \tag{29}$$

From (29), $\dot{V}(t) \leq 0$ for $\forall t \in [0, \infty]$, thus, $V(t)$ is decreasing or constant for $\forall t \in [0, \infty]$. Therefore, the following sufficient condition for (29) can be obtained

$$\dot{V} \leq -\frac{\alpha}{2}e^2 \quad \text{for} \quad \alpha\left(2k_1 + k_2 - \alpha^3\right) > \left(V(0) + \frac{1}{k_3}\ln(2) + \frac{1}{k_3}\right)^2 \tag{30}$$

where $V(0)$ is explicitly given as follows

$$\begin{aligned} V(0) &= \frac{1}{k_3}\ln\left(\cosh\left(k_3 r(0)\right)\right) + \frac{1}{2}e(0)^2 \\ &\leq |r(0)| + \frac{1}{2}e(0)^2 \end{aligned} \tag{31}$$

with $r(0)$ denoting the initial filtered tracking error signal. After utilizing the inequality of (31), the expression of (30) can be rewritten in the following manner

$$\dot{V} \leq -\frac{\alpha}{2}e^2 \quad \text{for} \quad \alpha\left(2k_1 + k_2 - \alpha^3\right) > \left(|\dot{e}(0)| + \alpha|e(0)| + \frac{1}{2}e(0)^2 + \frac{1}{k_3}\ln(2) + \frac{1}{k_3}\right)^2 \tag{32}$$

where (3) has been utilized.

From (13), (27), (32) and (18), $V(t)$, $r(t)$, $e(t)$, $\dot{e}(t) \in \mathcal{L}_\infty$ and $e(t) \in \mathcal{L}_2$. Based on the previous facts, Barbalat's Lemma [10] can then be employed to illustrate the result presented in (12). Standard signal chasing arguments can be employed to illustrate that all signals remain bounded during closed-loop operation.

5 Simulation Results

In order to illustrate the performance of the saturated controller of (5) and the learning based, disturbance estimator $\hat{\delta}(t)$ of (6), the system dynamics of (1) were simulated in Matlab's Simulink simulation package.The desired set-point position was selected as

$$x_d = 3 \times 10^{-2}\,(m), \tag{33}$$

and the periodic, nonlinear disturbance signal $\delta(t)$ is shown in Figure 2 (note that $\delta(t)$ is bounded and exhibits a period of $T = 1.0$ seconds). To illustrate the convergence abilities of the proposed control strategy, the target was set to the following initial position

$$x(0) = -0.03\ (m). \tag{34}$$

The control gains were adjusted to the following values to obtain the best performance

$$k_1 = 4 \qquad k_2 = 1 \qquad k_3 = 1 \qquad \alpha = 2 \qquad \delta_o = 2.25 \tag{35}$$

The target position signal $x(t)$ and the learning based estimation signal $\hat{\delta}(t)$ are shown in Figures 1 and 2, respectively.

6 Conclusions

In this paper, a saturated, force controller targeted for magnetic levitation systems that asymptotically regulates the position of a target mass in the presence of a nonlinear, bounded, periodic disturbance has been presented. The controller development does not require restrictions on the explicit structure of the disturbance other than an upper bound and a known period. In addition, the magnetic levitation actuator system achieves target position regulation despite the controller's limited actuation ability in that only an attractive force can be exerted onto the target mass (*i.e.*, the controller cannot generate a repulsive force on the target). Future research in the area of disturbance rejection for magnetic levitation systems will focus on the removing the requirement that the disturbance period to be known.

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Amplitudes and Frequencies in Linear SISO Uncertain Systems," *Proceedings of the American Control Conference*, Anchorage, Alaska, pp. 4015-4020, May 2002.

A Inequality Proof

After cancelling the resulting matching terms from the quadratic expansion, the inequality of (9) can be rewritten as follows

$$\left(sat_{\delta_0}\hat{\delta}(t)\right)^2 - 2\delta(t)\cdot sat_{\delta_0}\hat{\delta}(t) \leq \left(\hat{\delta}(t)\right)^2 - 2\delta(t)\cdot\hat{\delta}(t). \tag{36}$$

The term $\left[\left(\hat{\delta}(t)\right)^2 - 2\delta(t)\cdot\hat{\delta}(t)\right]$ is subtracted from both sides of (36), and the resulting expression is factored to obtain the following inequality

$$\left(sat_{\delta_0}\hat{\delta}(t) - \hat{\delta}(t)\right)\left(sat_{\delta_0}\hat{\delta}(t) + \hat{\delta}(t) - 2\delta(t)\right) \leq 0. \tag{37}$$

In order to verify the inequality of (37) (and thus the validity of (9)), the remainder of the proof is divided into the following three cases.

A.1 Case 1: $\left|\hat{\delta}(t)\right| \leq \delta_o$

Based on the condition that $\left|\hat{\delta}(t)\right| \leq \delta_o$, the function $sat_{\delta_0}\hat{\delta}(t)$ is evaluated as follows

$$sat_{\delta_0}\hat{\delta}(t) = \hat{\delta}(t); \tag{38}$$

hence, the inequality (37) is valid for the conditions of Case 1.

A.2 Case 2: $\hat{\delta}(t) > \delta_o$

Based on the condition that $\hat{\delta}(t) > \delta_0$ and from the definition of (7), the function $sat_{\delta_0}\hat{\delta}(t)$ is evaluated as

$$sat_{\delta_0}\hat{\delta}(t) = \delta_0 \tag{39}$$

which leads to the following two conditions

$$sat_{\delta_0}\hat{\delta}(t) - \hat{\delta}(t) \leq 0, \qquad sat_{\delta_0}\hat{\delta}(t) + \hat{\delta}(t) \geq 2\delta_0 \quad ; \tag{40}$$

therefore, the inequality of (37) is validated based on the fact that $\delta(t) \leq \delta_0$.

A.3 Case 3: $\hat{\delta}(t) < -\delta_o$

In a similar manner, the function $sat_{\delta_0}\hat{\delta}(t)$ is evaluated as follows for the condition when $\hat{\delta}(t) < -\delta_0$

$$sat_{\delta_0}\hat{\delta}(t) = -\delta_0 \tag{41}$$

which leads to the following two conditions

$$sat_{\delta_0}\hat{\delta}(t) - \hat{\delta}(t) \geq 0, \qquad sat_{\delta_0}\hat{\delta}(t) + \hat{\delta}(t) \leq -2\delta_0 \quad ; \tag{42}$$

therefore, the inequality of (37) is valid for the conditions of Case 3 based on the fact that $\delta(t) \geq -\delta_0$.

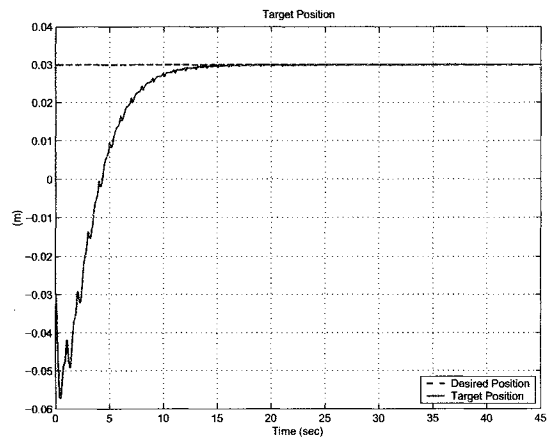

Figure 1: Target Position $x(t)$

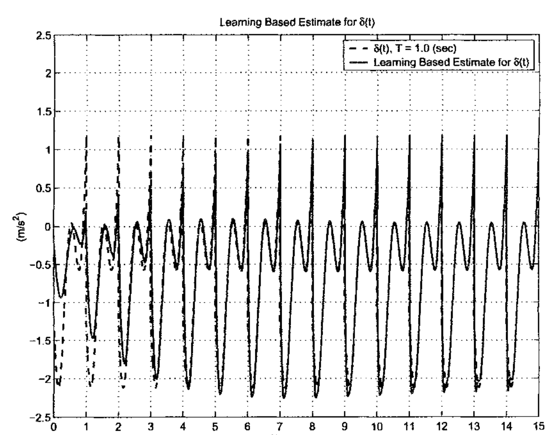

Figure 2: $\delta(t)$ and $\hat{\delta}(t)$